Roll No. .....

BT-4/M-24 44151

DISCRETE MATHEMATICS

PC-CS-202A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Prove that $[p \Rightarrow (q \Rightarrow r)] \Rightarrow [(p \Rightarrow q) \Rightarrow (p \Rightarrow r)]$ is a tautology. 5

   (b) Prove that : 5

   $$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

   (c) Prove the following proposition by PMI : 5

   $$1 + 2 + 3 + 4 \ldots\ldots\ldots\ldots + n = (n(n + 1))/2$$

2. Explain Principle of Inclusion and exlusion.A survey on a sample of 25 new cars was conducted to see which of three popular options, Air conditioning(A), radio(R), Power windows(P) were installed.The survey found that 15 had air conditioning, 4 had radio and power windows, 12 had radio, 3 had all three options, 5 had air conditioning and power windows, 2 had no options and 9 had air conditioning and radio. 15

(i) Only power windows

(ii) Only one of the options

(iii) Air conditioning and radio but not power windows.

(iv) Only radio.

Unit II

3. (a) Let A = {4, 6, 8, 10} and R = {(4, 4), (4, 10), (6, 6), (6, 8), (8, 10)}. Find the transitive closure using warshall's Algorithm and also write steps of the warshall's algorithm. **8**

(b) Define Relations. Let X = {1, 2, 3, 4, 5, 6} and R be a relation defined as $(x, y) \in R$, if and only if $x-y$ is divisible by 3. List the elements of Relation R. **7**

4. (a) Consider a set D45 = {1, 3, 5, 9, 15, 45}and let the relation $\leq$ be the relation (divides) be a partial ordering on D45 : **10**

(i) Determine GLB and LUB of B, B is subset of D45, where B = {9, 15, 45}

(ii) Determine GLB, LUB of B, B is subset of D45, where B = {1, 3, 5}

(iii) Draw Hasse diagram for D45.

Asymmetric relations with suitable examples. 5

Unit III

5. (a) Solve the recurrence relation $a_{r+2} - 5a_{r+1} + 6a_r = 2$ by using method of generating functions satisfying the initial conditions $a_0 = 1$ and $a_1 = 2$. 10

(b) How many people at least in a group of 85 people have same initials ? 5

6. (a) Describe types of functions with suitable examples. 10

(b) From a committee consisting of 6 men and 7 women, in how many ways can a committee be constructed, if committee consists of (i) 3 men and 4 women (ii) 4 members, which has at least one woman. 5

Unit IV

7. (a) Define the following terms with suitable examples : 10

(i) Monoid

(ii) Abelian Group

(iii) Ring Homomorphism

(iv) Group.

(b) Let G = {–1, 0, 1}, verify whether G forms a group under usual addition. **5**

8. (a) Consider an Algebraic system (Q, *), Q is a set of rational nos. and * is defined as $a*b = a + b - ab$ $\forall\ a, b \in Q$. Determine whether (Q, +) is a group. **10**

(b) Define a Semigroup. Write properties for a Semigroup. Explain with a suitable example. **5**