Roll No. .... Total Pages : 04

BCA/M-22 1874

MATHEMATICAL FOUNDATIONS-II

BCA-123

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all. Q. No. 9 is compulsory. Attempt *one* question from each Unit.

Unit I

1. (a) Show that $((\sim p)\wedge q)\wedge(q\wedge r)\wedge(\sim q)$ is a tautology. 8

(b) If p and q be any statements, then construct the truth table of the following statements : 8

(i) $(\sim p\vee q)\wedge(\sim p\wedge\sim q)$

(ii) $(p\Leftrightarrow\sim q)\Leftrightarrow(q\Rightarrow p)$

2. (a) Using Principle of Mathematical Induction, prove that for all $n \in$ N, 8

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.......+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

(b) Prove that $3^{2n+2}-8n-9$ is divisible by 64 for all $n \in$ N. 8

Unit II

3. (a) Show that the set $G = \{1, \omega, \omega^2\}$ is a group with respect to multiplication. Here 1, ω and ω^2 are cube roots of unity. **8**

(b) Find the order of the elements of the group $G = \{0, 1, 2, 3, 4\}$ under the binary operation 'multiplication modulo 5'. **8**

4. (a) Show that the set of matrices $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subring of the ring of 2×2 matrices with integral elements. **8**

(b) Prove that the set of real numbers is a field with respect to addition and multiplication. **8**

Unit III

5. (a) If $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$ then find X and Y. **8**

(b) Find the inverse of the matrix : **8**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

6. (a) Find the rank of the matrix : **8**

$$A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & -1 & 1 \\ 4 & -1 & 2 \end{bmatrix}$$

(b) Solve the following system of equations by Matrix method : **8**

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

Unit IV

7. Find the characteristic roots and the corresponding vectors for the following matrix : **16**

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

8. Verifly Cayley-Hamilton theorem and compute A^{-1} for the following matrix : **16**

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

Compulsory Question

9. (i) Define a Coset. **2**

(ii) Define Null Ring. **2**

(iii) Identify the quantifiers and write the negation of the statement : **2**

"Some diseases are curable and not infectious".

(iv) Define symmetric matrix with example. **2**

(v) Find the spectrum of the matrix : **2**

$$\begin{bmatrix} 5 & 0 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

(vi) Prove that $A - A^{\theta}$ is a Skew-Hermitian matrix if A is a square matrix. **2**

(vii) If a matrix A is singular, then prove that 'O' is a latent root of A. **2**

(viii) Let S = {0, 1, 2, 3, 4, 5}. Write composition table for S with respect to 'addition modulo 6'. **2**