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BCA/M-23 1866

MATHEMATICAL FOUNDATIONS-II

BCA-123

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) If p and q be any statement then construct the truth table $\sim(p \wedge q)$.

(b) Define subgroup.

(c) Define skew-symmetric matrix with example.

(d) Define prime ideal of a ring.

(e) State Cayley-Hamilton Theorem.

(f) Define Singular matrix.

(g) Define order of an element of a group.

(h) Construct a 2 × 2 matrix whose elements are given by $a_{ij} = i.j$. 8×2=16

Unit I

2. (a) Prove that $[(p \Leftrightarrow q) \wedge (q \Rightarrow r) \wedge r] \Rightarrow r$ is a tautology. 8

(b) Prove that $3^{2n+2} - 8n - 9$ is divisible by 64. 8

3. (a) Prove that $3^n > 2^n$ by Principle of Mathematical Induction for all $n \in \mathrm{N}$. 8

(b) Show that : 8

$$\sim (p \leftrightarrow q \equiv (\sim p) \leftrightarrow q \equiv p \leftrightarrow (\sim q).$$

Unit II

4. (a) Let G = {0, 1, 2, 3, 4}, find the order of the elements of the group G under the binary operation addition modulo 5. 8

(b) If every element of a group is its own inverse, then show that the group is abelian. 8

5. (a) Prove that intersection of the two subring is a ring. 8

(b) Let R be a ring of 2 × 2 matrices over integers. Let $S = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a, b \text{ integers} \right\}$, then S is a left ideal but not right ideal. 8

Unit III

6. (a) Find rank of the Matrix $\begin{bmatrix} 9 & 0 & 2 & 3 \\ 0 & 1 & 5 & 6 \\ 4 & 5 & 3 & 0 \end{bmatrix}$ by reducing it to Normal Form. 8

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, show that : 8

$$A^3 - 23A - 40I = 0.$$

7. (a) Solve using rank method : 8

$$x + y + z = 0$$
$$2x - 3y + z = 9$$
$$x - y + z = 0.$$

(b) Solve : 8

$$x - y + z = 0$$
$$x + 2y - z = 0$$
$$2x + y + 3z = 0.$$

Unit IV

8. Find eigen values and eigen vectors of the Matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}.$$ 16

9. Verify Cayley-Hamilton Theorem for the Matrix

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, and hence find its inverse. 16