Roll No. ..... **Total Pages : 2**

BCA/M21 1889

MATHEMATICAL FOUNDATIONS-II

Paper–BCA-123

Time allowed : 3 Hours **Maximum Marks : 80**

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question No. **9** is compulsory. All questions carry equal marks.

UNIT-I

1. (i) Show that : $[(p \to q) \ (q \to r)] \to (p \to r)$ is a tautology. 8

   (ii) Construct the truth table of the following statement : 8

   (a) $(p \Leftrightarrow \sim q) \Leftrightarrow (q \Rightarrow p)$ (ii) $(p \wedge q)$ ϖ $\sim(p$ ϖ $q)$.

2. (i) Prove by the principle of Mathematical Induction that the sum of first n natural number is $\frac{n(n+1)}{2}$. for all $n \in N$. 8

   (ii) For all $n \in N$, show that $11^{n+2} + 12^{2n+1}$ is divisible by 133. 8

UNIT-II

3. (i) Show that the set $G = \{-1, 1, -i, i\}$ is a group with respect to multiplication. 8

   (ii) Let $G = \{0, 1, 2, 3, 4\}$. Find the order of the elements of the groups G under the binary operation 'addition modulo 5.' 8

4. (i) Prove that the set of Rational numbers is a field with respect to addition and multiplication. 8

   (ii) Prove that the necessary and sufficient conditions for a non-empty subset S of ring R to be a subring of R are

   (a) a, b ∈ S ⇒ a – b ∈ S (b) a, b ∈ S ⇒ a – b ∈ S. 8

UNIT-III

5. (i) Find the inverse of the matrix : $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 8

   (ii) Find X and Y if $2X + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ and $X - 2Y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$. 8

6. (i) Find the rank of the following matrix : 8

$$\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$$

(ii) Using matrix method, solve the following system of equation : 8

$x + y + z = 6$

$x - y + x = 2$

$2x + y - z = 1.$

UNIT-IV

7. Find the characteristics roots and the corresponding vectors for the following matrix : 16

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

8. Verify Cayley Hamilton theorem and find A^{-1} for the matrix : 16

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

Compulsory Question

9. (i) Identify the quantifiers and write the negation of the statements "There exists a capital for every state in India." 2

(ii) Define Normal Subgroups. 2

(iii) Define Ring with unity. 2

(iv) Define Skew-symmetric matrix with example. 2

(v) Find the Spectrum of the matrix : 2

$$\begin{bmatrix} 2 & 7 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

(vi) Prove that 'O' is a latent root of a matrix A if A is singular. 2

(vii) If A is a square matrix then show that $A + A^{\theta}$ is Hermitian. 2

(viii) Write composition table for S with respect to multiplication modulo σ where S = {0, 1, 2, 3, 4, 5}. 2