GSE/D-22 1163

MATHEMATICAL FOUNDATIONS-I

BCA-113

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Draw Venn diagram for $A \cup B$.

(b) Define Lattices.

(c) Evaluate :

$$\lim_{x \to 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}.$$

(d) Find order and degree of the differential equation

$$\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{1/3} = xy.$$

(e) Solve the differential equation :

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0.$$

Unit I

2. (a) Prove that :

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(b) Show that if R_1 and R_2 are equivalence relation on A then $R_1 \cap R_2$ is an equivalence relation.

3. (a) A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drown from the box, if at least one black ball is to be included in the draw ?

(b) If $f(x,y,z)=(x\cup y)\wedge(x\cup y')\wedge(x'\cup z)$ be a given Boolean function, determine its DN form.

Unit II

4. (a) Prove that limit of a function at a point, if exists, is unique.

(b) Find the value of a if the function f given by

$$f(x)=\begin{Bmatrix} 2x-1, & 2<x \\ a \quad , & x=2 \\ x+1 \ , & x>2 \end{Bmatrix}$$ is continuous at $x = 2$.

5. (a) Find $\frac{dy}{dx}$ if $y=\frac{\sqrt{x}(x+4)^{3/2}}{(4x-3)^{4/3}}$.

(b) Find $\frac{dy}{dx}$ if $y=(\sqrt{x})^{x}+(x)^{\sqrt{x}}$.

(c) If $= x^x$, show that :

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$$

Unit III

6. (a) Form the differential equation of the equation $(x-a)^2 + (y-b)^2 = r^2$ by eliminating the arbitrary constants a and b.

(b) Solve the differential equation :

$$y\sqrt{1-x^2}\,dy + x\sqrt{1-y^2}\,dx = 0.$$

7. (a) Solve the differential equation :

$$(x^2 - y^2)dx - xydy = 0.$$

(b) Solve the differential equation :

$$(x^2 + y^2 + 2x)dx + 2ydy = 0.$$

Unit IV

8. (a) Solve the differential equation :

$$\frac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}.$$

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x.$$

9. (a) Solve the differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x.$$

(b) Determine the curve in which the length of the sub-normal is proportional to the square of the ordinate.