Roll No........................

BT-3/ D-21: 43135

BS-205 A : Mathematics-III

[Max. Marks: 75

Time: 3 Hours]

Note: Attempt any five questions.

1 (a) Test the convergence of $\displaystyle\sum_{n=1}^{\infty}\left(\frac{(n+1)(n+2)}{n^2\sqrt{n}}\right)$

(b) Discuss the convergence or divergence of the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\cdots \qquad x>0$$

2 (a) Expand $f(x)=x\sin x$ as a Fourier series in $(0,2\pi)$.

(b) Find the half -range sine series for $f(x)=x(\pi-x)$ in the

interval $(0,\pi)$ and deduce that

$$\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+--=\frac{\pi}{32}$$

3 (a) Solve $(x^2y-2xy^2)\,dx-(x^3-3x^2y)\,dy=0$ using exact
differential equation.

(b) Solve the differential equation

$$x^2\left(\frac{dy}{dx}\right)^2+xy\frac{dy}{dx}-6y^2=0$$

4 (a) Solve $\dfrac{d^3y}{dx^3}-6\dfrac{d^2y}{dx^2}+11\dfrac{dy}{dx}-6y=e^{2x}$

(b) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$$

5 (a) Change the order of integration in the interval:

$$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2(ax)^{\frac{1}{2}}}dy\,dx$$

(b) Show that area between the parabolas $y^2=4ax$ and $x^2=4ay$ is

$$\frac{16}{3}a^2$$

6 (a) Evaluate

$$\int\int_{1}^{e\,log\,y}\int_{1}^{e^{x}} log\,z\,dz\,dx\,dy$$

(b) Find the volume of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

7 (a) If $\vec{r} = x\imath + y\jmath + z\hat{k}$. show that $\Delta r^n = nr^{n-2}\vec{r}$

(b) Prove that $\nabla^2 f(r) = f''(r) + \dfrac{2}{r}f'(r)$

8 (a) Evaluate the line integral $\int_c (x^2 + xy)\,dx + (x^2 + y^2)\,dy$.

where C is the square formed by the lines $x = \pm 1, y = \pm 1$

(b) Verify Green's Theorem for $\oint_c [(xy + y^2)\,dx + x^2\,dy]$.

where c is bounded by $y = x$ and $y = x^2$.