Roll No. .... **Total Pages : 04**

BT-3/D-20 **43135**

MATHEMATICS-III

BS-205A

Time : Three Hours] [Maximum Marks : 75

Note : All questions in Part A and Part B are compulsory. Attempt any *four* questions from Part C, selecting *one* question from each Unit.

Part A

1. (a) Determine the following series converges or diverges $\sum_{n=2}^{\infty} \frac{1}{n^5 - n^2 - 1}$.

(b) Solve $3y' + xy = xy^{-2}$.

(c) Find the solution of :

$$\left(D^2 + 4D + 4\right) y = 5 \cos x.$$

(d) Evaluate the integral :

$$\int_0^1 \int_0^x \int_0^{1+2x+3y} f(x, y, z)\, dx dy dz, \text{ where } f(x, y, z) = 5.$$

(e) Evaluate Curl of $e^{xyz}(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ at the point (1, 2, 3).

5×3=15

Part B

2. Determine whether the series converge :

(a) $\sum_{n=1}^{\infty} \frac{n}{e^n}$.

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{5^n}$. **5**

3. Solve : **5**

$$\left(y + \sqrt{x^2 + y^2}\right) dx - x dy = 0,\ y(1) = 0.$$

4. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$. **5**

5. Calculate $\nabla^2 f$ when $f = 3x^2 z - y^2 z^3 + 4x^3 y + 2x - 3y - 5$ at the point (1, 1, 0). **5**

Part C

Unit I

6. (a) Define Cauchy first root test for sequence. Also check the convergence of $<a_n>$, where

$$a_n = \left(\frac{n^3 + n}{n + 5}\right)^{\frac{1}{n}}.$$ **5**

(b) Series $\sum \frac{1}{n!}$ converges or diverges ? Justify. **5**

7. Expand $f(x) = x \sin x$ as a Fourier series in $(0, 2\pi)$. **10**

Unit II

8. (a) Find the general solution and singular solution of $y = \sin(y - xp)$. **5**

(b) Solve $y\left(2x^2 - xy + 1\right) dx + (x - y) dy = 0$ using exact differential equation. **5**

9. (a) Solve : **5**

$$\left(D^2 - 4D - 5\right) y = e^{2x} + 3 \cos(4x + 3)$$

(b) Solve the following differential equation using method of variation of parameter $\left(D^2 - 2D\right) y = e^x \sin x$. **5**

Unit III

10. Evaluate $\iint \sqrt{\frac{1 - x^2 - y^2}{1 + x^2 + y^2}}$ over the positive quadrant of the circle $x^2 + y^2 = 1$. **10**

11. Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ay$. **10**

Unit IV

12. Show that $\nabla \times \left(\nabla \times \bar{A}\right) = \nabla\left(\nabla \cdot \bar{A}\right) - \nabla^2 \bar{A}$ for any vector function $\bar{A}$. **10**

13. Verify the Stokes theorem for $\bar{A} = y^2 \boldsymbol{i} + xy \boldsymbol{j} + xz \boldsymbol{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$. **10**