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BT-3/D-23

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MATHEMATICS-III

BS-205A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Section. All questions carry equal marks.

Section A

1. Examine the convergence of the series :

   (i) $\dfrac{1}{2\sqrt{1}} + \dfrac{x^2}{3\sqrt{2}} + \dfrac{x^4}{4\sqrt{3}} + \dfrac{x^6}{5\sqrt{4}} + ...\infty$

   (ii) $\dfrac{1}{2} + \dfrac{2}{3}x + \left(\dfrac{3}{4}\right)^2 x^2 + \left(\dfrac{4}{5}\right)^3 x^3 + ...\infty$

2. Prove that $x^2 = \dfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \dfrac{\cos nx}{n^2}$, $-\pi < x < \pi$.

   Hence show that :

   $$\sum \dfrac{1}{n^2} = \dfrac{\pi^2}{6}.$$

P.T.O.

Section B

3. Solve :

$$\frac{2x}{y^3} dx + \frac{(y^2 - 3x^2)}{y^4} dy = 0.$$

4. Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$ using the method variation of parameter for finding the particular integral.

Section C

5. Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy \, dx$.

6. Evaluate $\iint_D (x + 2y) dx \, dy$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Section D

7. For the function $\phi(x, y) = \frac{x}{x^2 + y^2}$, find the magnitude of the directional derivative along a line making an angle 30 with the positive x-axis at $(0, 2)$.

8. State Green's Theorem for a plane and verify the same for $\int_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$, where C is the boundary of the region bounded by $x \geq 0$, $y \leq 0$ and $2x - 3y = 6$.