

Roll No. ....

Total Pages : 03

BT-1/D-22

41040

MULTI-VARIABLE CALCULUS AND  
LINEAR ALGEBRA  
BS-135A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

**Unit I**

1. (a) Prove the following : 7.5

$$\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx \times \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{4\sqrt{2}}.$$

- (b) Find the volume of the solid generated by revolution of the plane area bounded by  $y^2 = 16x$  and  $y = 4x$  about the  $x$ -axis. 7.5

2. (a) Discuss the applicability of Rolle's theorem to the function  $f(x) = |x|$  in  $[-1, 1]$ . 7.5

- (b) Evaluate the limit of the function : 7.5

$$\text{Lt}_{x \rightarrow 0} (\text{cosec } x)^{\frac{1}{\log x}}.$$

## Unit II

3. (a) Test the convergence of the  $\sum \frac{n^3 + a}{2^n + a}$ . 7.5

(b) Discuss the convergence of the series : 7.5

$$\frac{1^2}{2^2} + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \cdot \frac{5^2}{6^2} + \dots$$

4. (a) Find the Fourier series to represent : 7.5

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

(b) Obtain the Fourier series of the function : 7.5

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2. \end{cases}$$

## Unit III

5. (a) Expand  $\cos x$  in powers of  $\left(x - \frac{\pi}{2}\right)$ , and hence find the value of  $\cos 91^\circ$  corrected up to four decimal places. 7.5

(b) If  $u = f(r)$  where  $r^2 = x^2 + y^2$ , prove that : 7.5

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

6. (a) If  $(\cos x)^y = (\sin y)^x$ , find  $\frac{dy}{dx}$ . 7.5

(b) A thin closed rectangular box is to have one edge equal to twice the other, and a constant volume  $72 \text{ m}^3$ . Find the least surface area of the box. 7.5

### Unit IV

7. (a) By applying the Gauss Jordan Method, find the

inverse of the matrix  $\begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$ . 7.5

(b) If matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ , find the matrix

represented by : 7.5

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 6A^2 - 3A + 2I.$$

8. Find the eigen values and the corresponding eigen vectors

of the matrix given by  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ . 15