Roll No. .... **Total Pages : 04**

BT-3/D-20 **43080**

DISCRETE STRUCTURES

CSE-201N/IT-209N

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) In a survey of 60 people, it was found that : 25 read Newsweek magazine, 26 read Time, 26 read Fortune, 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, 3 read all three magazines. **10**
   (i) Find the number of people who read at least one of the three magazines.
   (ii) Find the number of people who read exactly one magazine.
   (iii) Find the number of people who read Newsweek and Time but not all three magazines.

   (iv) Find the number of people who read Newsweek and Fortune but not all three magazines.
   (v) Find the number of people who read Fortune and Time but not all three magazines.
   (vi) Find the number of people who read only Newsweek.
   (vii) Find the number of people who read only Time.
   (viii) Find the number of people who read only Fortune.
   (ix) Find the number of people who read no magazine at all.

   Also draw a Venn diagram of the above problem.

   (b) Determine whether or not $\sim p \leftrightarrow (p \vee \sim p)$ is a tautology or contradiction. **5**

2. (a) Prove that $(ab)^n = a^n b^n$ is true for every natural number n. **7**
   (b) What are normal forms ? Discuss its various types using suitable examples. **8**

Unit II

3. (a) Prove that $(D_{30} \leq)$ is a lattice. Also draw a hasse diagram of D_{30}. **7**

   (b) Let $\sum = \{a, b\}$. Define a relation R on $\sum^*$ as : xRy if x is a prefix of y. Is R a partial order ? **8**

4. (a) Write down the Warshall's algorithm for finding the shortest path. Explain the algorithm using suitable examples. **10**
   (b) Let A = {0, 1, 2, 3} and let r = {(0, 0), (1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (3, 2), (2, 3), (3, 1), (1, 3)}. **5**
   (i) Show that r is an equivalence relation on A.
   (ii) Let a belongs to A and define $c(a)$ = {b belongs to A | a r b}, c (a) is called the equivalence class of the elements a under r. Find c (a) for each element a belonging to A. **5**

Unit III

5. (a) Using generating functions, solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, where $a_0 = 2$ and $a_1 = 3$. **7**
   (b) Prove that 'A function f : A → B is invertible if and only if both one-to-one and onto'. **8**

6. (a) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there ? **10**
   (b) State and prove Pigenhole principle. **5**

Unit IV

7. (a) Define a semigroup and a group and prove that a semi-group G is a group if and only if the equations $ax = b$ and $ya = b$ has solutions in G for arbitrary $a, b \in G$. **7**
   (b) Define homomorphism and its properties. Check whether $\theta : Z_5 \rightarrow Z_2$ is defined by $\theta(n) = 0$ if n is even and $\theta(n) = 1$ if n is odd. **8**

8. (a) Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7. **8**
   (i) Find the multiplication table of G.
   (ii) Find inverse of 2, 3, 6.
   (iii) Find the orders and subgroups generated by 2 and 3.
   (iv) Is G cyclic ?
   (b) Prove that H, a subset of group [G; *] is a sub-group. **7**